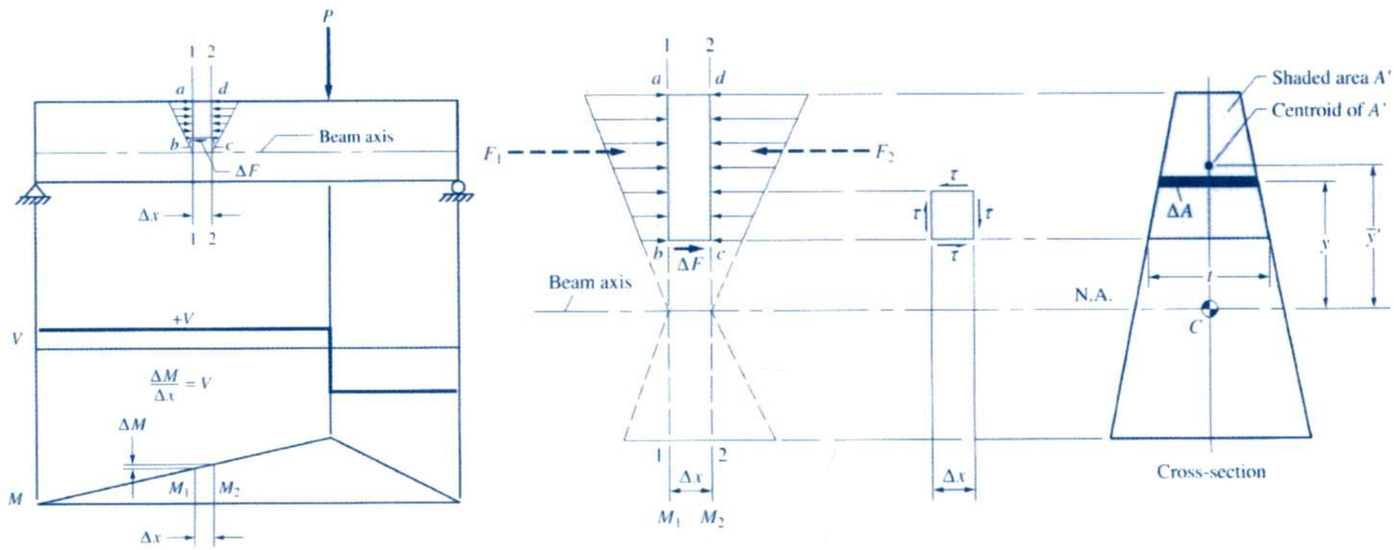
14-5

Shear Stress Formula For Beams

- In addition to withstanding applied bending moments, beams also need to have the ability to resist shear forces.
- Shear forces along the beam are also generated by applied loads and actually try to "rip the beam apart." If a beam cannot resist the shear forces a failure occurs.
- Typically, shear is not the limiting factor in design, but it must still be checked. For materials weaker in shear (concrete, wood) it may be the control parameter for the design.

Derivation of Shear Stress Formula



$$F_1 = \Sigma \sigma \Delta A = \Sigma \frac{M_1 y}{I} \Delta A = \frac{M_1}{I} \Sigma y \Delta A$$

$$F_2 = \Sigma \sigma \Delta A = \Sigma \frac{M_2 y}{I} \Delta A = \frac{M_2}{I} \Sigma y \Delta A$$

By horizontal Equilibrium $[\Sigma F_y = 0]$

$$F_1 + \Delta F - F_2 = 0 \Rightarrow \Delta F = F_2 - F_1$$

$$\Delta F = F_2 - F_1 = \frac{M_2 - M_1}{I} \Sigma y \Delta A = \frac{\Delta M}{I} \Sigma y \Delta A$$

Shear stress

$$\tau = \frac{V}{A_s} = \frac{\Delta F}{t \Delta x} = \frac{\Delta M}{\Delta x} \frac{\Sigma y \Delta A}{I t}$$

$$\frac{\Delta M}{\Delta x} = V$$

and denote $\Sigma y \Delta A$ by Q

$$\boxed{\tau = \frac{VQ}{It}} \qquad (14\text{-}10)$$

The shear stress at any location on a cross-section of a beam (and at any point along the beam's length) can be found through the general shear stress equation.

$$\tau = \frac{VQ}{It}$$

Where,

τ = the shear stress at a point in a given section of the beam

V = the shear force at the given section

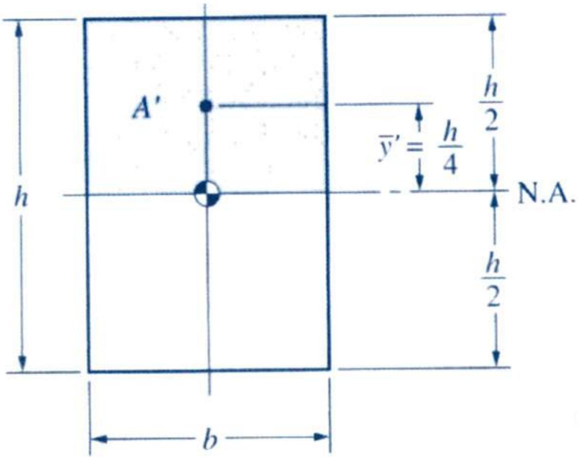
Q = the first moment of the area A′ about the neutral axis, $Q = A'\bar{y}'$

- A′ = the part of the areea in the cross-section above (or below) the horizontal line where the shear stress is to be calculated.
- $\bar{y}'$ = the distance from the neutral axis to the centroid of the area A′

I = the moment of inertia of the entire section with respect to the neutral axis (the same I as in the flexure formula)

t = the width of the cross-section at the horizontal line where the shear stress is being calculated

Maximum Shear Stress in a Rectangular Section



The maximum value of Q occurs at the neutral axis

$$Q = A'\bar{y}' = \left(b \times \frac{h}{2}\right)\left(\frac{h}{4}\right) = \frac{bh^2}{8}$$

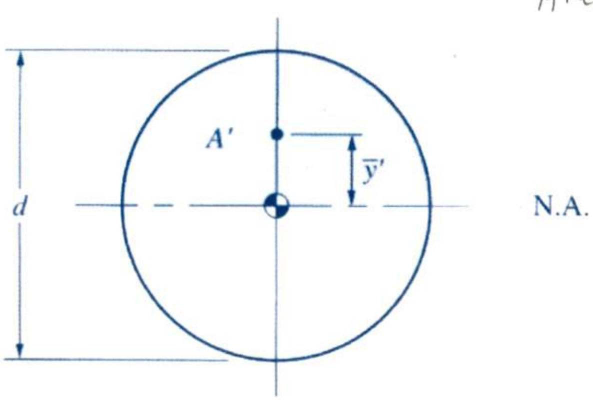
The maximum shear stress,

$$\tau_{MAX} = \frac{VQ}{It} = \frac{V\left(\frac{bh^2}{8}\right)}{\left(\frac{bh^3}{12}\right)(b)} = \frac{3V}{2bh}$$

and

$$\boxed{\tau_{MAX} = 1.5\frac{V}{A}} \qquad (14\text{-}11)$$

Maximum Shear Stress in a Circular Section



Area of a semicircle (Table 7-2)

$$A' = \frac{\pi d^2}{8} \qquad \bar{y}' = \frac{2d}{3\pi}$$

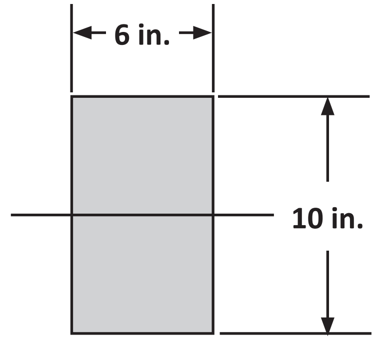
$$Q = A'\bar{y}' = \left(\frac{\pi d^2}{8}\right)\left(\frac{2d}{3\pi}\right) = \frac{d^3}{12}$$

$$\tau_{MAX} = \frac{VQ}{It} = \frac{V\left(\frac{d^3}{12}\right)}{\left(\frac{\pi d^4}{64}\right)(d)} = \frac{16V}{12\left(\frac{\pi d^2}{4}\right)} = \frac{4V}{3A}$$

$$\boxed{\tau_{MAX} = \frac{4V}{3A}} \qquad (14\text{-}12)$$

Example 1

Calculate the shear stress for the beam shown
(a) at the neutral axis
(b) 3 in. away from the neutral axis
(c) at the outside surface



The shear force on the beam is, V = 50 kips

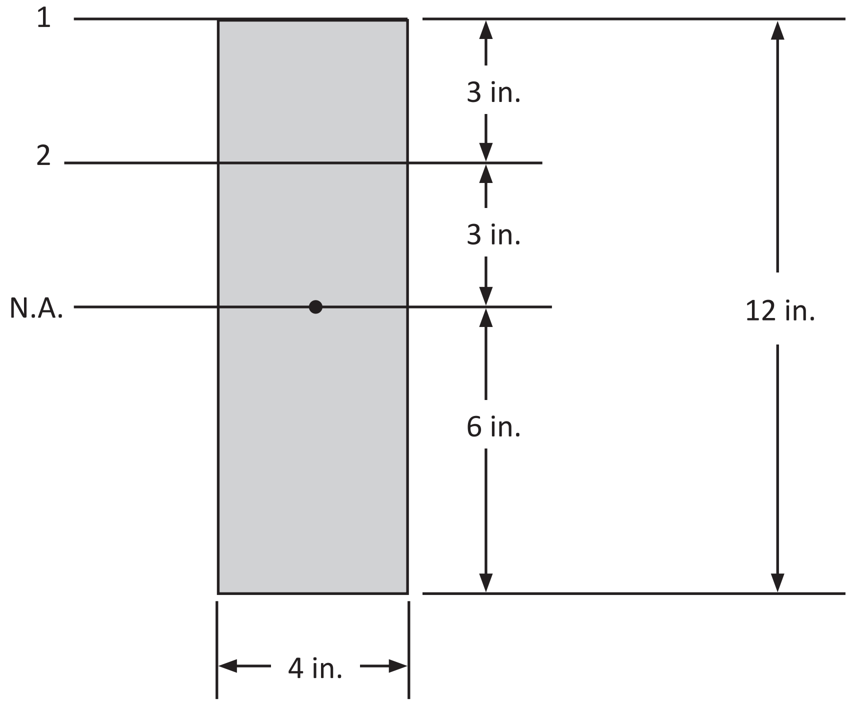
Solution.

Example 2

Calculate the shear stresses at the neutral axis and at the midpoint of the flange in a W 16 x 77, if the shear force is 100 kip.

Solution.

Example 14-6 (Changed to U.S. System of Units)
The simple beam is subjected to a concentrated load P = 4500 lb at the midspan. The beam has the rectangular section shown. Determine the shear stresses at points along line 1, line 2, and the neutral axis. Sketch the shear stresses distribution in the section.



Solution.

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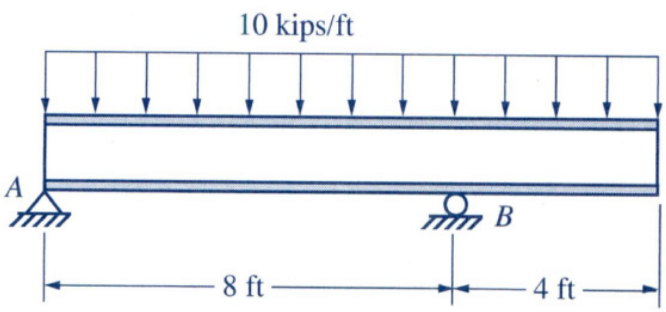
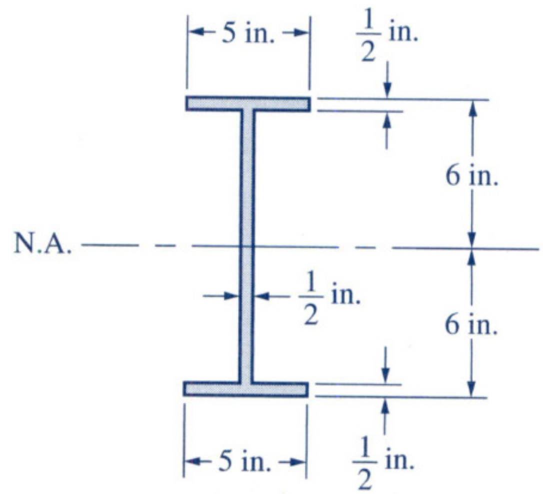
Example 14-7

The overhanging beam in Fig. E14-7(1) is subjected to a uniform load as shown. The beam is built of three steel plates welded together to form an integral section, as shown in Fig. E14-7(2). Such a built-up section is called a plate girder. (a) Calculate the maximum shear stress and indicate this stress on a rectangular element at the point where it occurs. (b) At the section where the maximum shear stress occurs, calculate the shear stress at the junction of the flange and the web. (c) Plot the distribution of the shear stress for the section where the maximum shear stress occurs.





Solution.